Image Analysis for Cosmology :
Results from the GREAT10 Star Challenge

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ABSTRACT

We present the results from the first public blind PSF reconstruction challenge, the GRavitational lEnsing Accuracy Testing 2010 (GREAT10) Star Challenge. Reconstruction of a spatially varying PSF, sparsely sampled by stars, at non-star positions is a critical part in the image analysis for weak lensing where inaccuracies in the modelled ellipticity e and size R^2 can impact the ability to measure the shapes of galaxies. This is of importance because weak lensing is a particularly sensitive probe of dark energy, and can be used to map the mass distribution of large scale structure. Participants in the challenge were presented with 27,500 stars over 1300 images subdivided into 26 sets, where in each set a category change was made in the type or spatial variation of the PSF. Thirty submissions were made by 9 teams. The best methods reconstructed the PSF with an accuracy of $\sigma(e) \approx 2.5 \times 10^{-4}$ and $\sigma(R^2)/R^2 \approx 7.4 \times 10^{-4}$. For a fixed pixel scale narrower PSFs were found to be more difficult to model than larger PSFs, and the PSF reconstruction was severely degraded with the inclusion of an atmospheric turbulence model (although this result is likely to be a strong function of the amplitude of the turbulence power spectrum).

Subject headings: Cosmology: observations, Methods: data analysis, Atmospheric effects, Techniques: image processing

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1. Introduction

In this paper we present the results from the GRavitational lEnsing Accuracy Testing 2010 (GREAT10) Star Challenge. GREAT10 was an image analysis challenge for cosmology that focused on the task of measuring the shapes of distant galaxies. Light from distant galaxies is deflected during its journey to us via gravitational lensing, and the images appear distorted into characteristic patterns (Hu 1999; Bartelmann & Schneider 2001). The amount of distortion depends on the intervening distribution of matter (including dark matter) and the geometry of spacetime (which is currently governed by dark energy). Such measurements thus probe directly the invisible dark sector and the fundamental nature of gravity — see reviews by Albrecht et al. (2001); Réfrégier (2003); Hoekstra & Jain (2008); Massey, Kitching & Richard (2010); Weinberg et al. (2012).

All real imaging data are necessarily seen after convolution with (i.e. blurring by) a telescope's Point Spread Function (PSF). The PSF arises from the finite aperture of the telescope, charge diffusion within digital detectors, any imperfect elements along the optical path, and turbulence in the Earth's atmosphere (unless the telescope is in space). This increases the size of faint galaxies, and can spuriously change their ellipticity by an order of magnitude more than gravitational lensing (Bernstein & Jarvis 2002; Hoekstra 2004; Paulin-Henriksson et al. 2008, 2009; Massey et al. 2012). To recover the shape of the galaxy after only cosmological effects, it is necessary to (1) model the PSF and (2) somehow correct for its effect on the images of galaxies. The second half of this task has been widely addressed by teams analysing individual surveys and, as a vital community effort, through the public Shear TEsting Programme (STEP) (Heymans et al. 2006; Massey et al. 2007), the GRavitational lEnsing Accuracy Testing (GREAT) galaxy challenges (Bridle et al. 2010; Kitching et al. 2012b) and the Mapping Dark Matter challenge (Kitching et al. 2012c). The first task (modelling the PSF) has so far only been investigated internally within teams (e.g. Bacon et al. 2003; Hoekstra, Yee & Gladders 2004; van Waerbeke et al. 2005; Rhodes et al. 2007; Schrabback et al. 2010; Hoekstra 2004; van Waerbeke et al. 2005; Rowe 2010; Jarvis & Jain 2005). Here we present the results of the first blind, public trial of methods to model and interpolate the PSF of a typical astronomical telescope.

The PSF in an astronomical image can be measured from stars that happen to fall inside the field of view. Stars are so small that they are intrinsically point-like, and adopt the size and shape of the telescope's PSF. However, the PSF typically varies across the field of view, and stars only sparsely cover the extragalactic sky (Jarvis & Jain 2005; Jain, Jarvis & Bernstein 2006; Heymans et al. 2012; Chang et al. 2012). It is therefore necessary to model the shapes of stars, then interpolate their shapes to the locations of galaxies (where there is necessarily not a bright star, because otherwise the galaxy could not be seen). In practice the PSF also varies as a function of the wavelength of observed light, due to diffraction, reflection and transmission effects in the telescope optics, filters and CCDs and so must also be interpolated from the colours of the stars to the colours of the galaxy (Cypriano et al. 2010; Voigt et al. 2011; Plazas & Bernstein 2012). Colour dependence is an important second order effect but in this paper we do not address this, focussing only on the primary changes in PSFs.

We simulated the spatial variation in the PSF of generic but realistic ground-, balloon-, and space-based telescopes (Kitching et al. 2012a, and see www.greatchallenges.info). We realised a large suite of sparse stellar fields in these different observing regimes, and publicly released *most* of the star images. Entrants were asked to then reconstruct the images of the missing stars on a pixel grid, at pre-defined locations. The performance of each entry was measured in real time using a single number, 'quality factor', which was designed to provide a crude ranking such that

it could not be reverse-engineered to reveal the full solutions. In this paper, we analyse in detail the quantitative performance of 12 distinct algorithms submitted to model and interpolate the simulated PSFs. In particular we quantify how well the ellipticity and size of a spatially varying PSF can be reconstructed in a blind challenge.

This paper is organised as follows. In Section 2, we describe the simulations and competition in detail. In Section 3, we present results. We discuss and conclude in Section 4.

2. Method

In this Section we describe the simulations and the competition. For a full exposition of the background of the Star Challenge see Kitching et al. (2012a).

2.1. Simulation Structure

In the simulations we aimed to generate simplified representations of possible observing scenarios and telescopes, such that through analysis we could make general statements about how methods perform in a coarse-grained sense in each of these categories.

The simulations contained two possible types of PSF function: a Moffat function (Moffat, 1969) and an Airy disk, parameterized by a FWHM size. To simulate diffraction spikes caused by obscuration of the telescope pupil the intensity distributions of these functions were optionally combined with single-slit diffraction intensity patterns, approximating the effects of rectangular obscurations in the pupil plane as would be caused by struts supporting a secondary mirror. The dimensions of these single slit obscurations were chosen to produce simulated PSFs of reasonable realism on visual inspection; for the Airy disk this corresponded to a strut obscuration of width 4% the pupil diameter. The configurations chosen for these diffraction spike patterns were a 'plus-sign' four-fold symmetric mask +, or an 'asterisk-sign' six-fold symmetric pattern $*$[1]. The combined pattern was then given a linear coordinate shear to create elliptical PSF patterns, and the PSF spatial variation for any image then contained three components, similar to the PSF described in Kitching et al. (2012b: Appendix C, where we refer the reader to Figures that show the PSF variation):

- **Static Component.** These were spatially constant across the image and consisted of i) a Gaussian smoothing kernel that added to the PSF size, this had a variance of 0.1 present in all images, ii) a static additive ellipticity component of 0.05 in $e_{1,\mathrm{PSF}}$ and $e_{2,\mathrm{PSF}}$, to simulate tracking error (e_1 and e_2 are defined in Section 2.3). Details are explained in Kitching et al. (2012b).

- **Deterministic Component.** This was to simulate the impact of the telescope on the spatially varying PSF size and ellipticity. We used the Jarvis et al. (2008) model with fiducial parameters given in Kitching et al. (2012b) ($a_0 = 0.014$, $a_1 = 0.0005$, $d_0 = -0.006$, $d_1 = 0.001$, $c_0 = -0.010$), which is dominated by primary astigmatism (a_0), primary defocus (d_0) and coma (c_0).

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[1]We use the term 'mask' to label such configurations, but we remind that reader that for the $*$ pattern a telescope would only have 3 struts arranged in a trefoil shape - it is the slit diffraction that results in six spikes in the images.

- **Random Component.** To simulate the random turbulent effect of the atmosphere we additionally included a random Gaussian field in some images in the ellipticity only, with a Kolmogorov-like power spectrum of $C_\ell = \ell^{-11/6}$. In fact subsquent to the formulation of this challenge, and launch in 2010, Heymans et al. (2012) found that $C_\ell \propto \ell^{-11/3}$, the exact power was not know accurately beforehand hence we refer to the $C_\ell \propto \ell^{-11/6}$ as Kolomogorov-like; this is approximately similar to short exposures from a ground-based observatory for a Moffat PSF, or balloon-based if an Airy PSF is used. We note that the amplitude of the power is also very high, corresponding to exposures of $\simeq 1$ second (see Heymans et al., 2012): we leave an investigation into the impact of varying amplitudes of Kolmogorov power to future work.

The integration of the PSF intensity distribution onto square pixels was achieved by multiplication with a Sinc function in Fourier space (equivalent to convolution with a square boxcar function in real space), followed by sampling at the locations of pixel centres.

2.2. Data Structure

The simulation was designed within the constraint that both the download size of the simulation and the upload size of the submissions should be manageable (we limited the download size to 50 Gb). Participants were provided with FITS (Wells et al. 1981) images containing 'known-stars' that were delta functions convolved with a spatially varying PSF. Each star within each image was embedded in a postage stamp of 48x48 pixels, and to reduce the size of the images there was no noise in between postage stamps. Participants were then asked to submit a 2D image of the reconstructed PSF at positions in between the known-stars; these positions were provided as a catalogue of 'asked-star' positions. Participants were asked to submit FITS cubes of the reconstructed PSFs (the x and y dimensions representing the 2D image and the z dimension varying the asked-star positions).

For each image 1000 asked-stars were required. The images were subdivided into 26 sets of 50 images where in each set the the properties of the spatial variation, telescope and static components of the PSF were kept statistically constant, but each had a different realisation of any random component, and each also had the asked- and known-star positions varying. The properties of each set are summarised in Table 1. One aspect to note is that when varying the size of the PSF in the total flux was kept constant for each profile; with an integrated signal to noise of 100.

2.3. Competition Structure

The competition started in December 2010 and ran for 9 months until September 2011; this was concurrent with the GREAT10 Galaxy challenge (Kitching et al. 2012a,b). As stated previously the total simulation size was ~ 50 Gb and the total size of the uploaded submissions was ~ 1 Gb (we allowed participants to tar, zip or FITS-compress[2] submissions to reduce size). Data and example code were provided online for participants[3].

The two parameters of the PSF that most directly impact the ability to interpret observations of galaxies are the ellipticity and the size of the PSF; any residual difference between the ellipticity or size of true PSF, and the respective quantities of the modelled PSF at any particular position, will result in errors and biases in parameters assigned to any galaxy at that position. Weak gravitational lensing is particularly sensitive to these types of error (Massey et al. 2012; Paulin-Henriksson et al. 2008, 2009). The ellipticity and size are defined here using the second order brightness moments of the image as

$$q_{ij} = \frac{\sum_p w_p I_p (\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\sum_p w_p I_p}, \quad i, j \in \{1, 2\}, \tag{1}$$

where the sums are over pixels, I_p is the flux in the pth pixel and θ is a pixel position ($\theta_1 = x_p$ and $\theta_2 = y_p$). In order to regularise the results with regard to the impact of noise but not to constrain the interpretation to compact objects in the postage stamp, we include a weight function w_p chosen to be a broad Gaussian with a width of 24 pixels (we leave an investigation of how results vary as a function of weight for future work). These are almost unweighted quadrupole moments in this respect, and as a result, smooth analytical functions may be favoured compared to models that try to reproduce details in the wings of the PSF. The weighted ellipticity (or technically the 'polarisability') for a PSF in complex notation is defined as

$$e = \frac{q_{11} - q_{22} + 2\mathrm{i}q_{12}}{q_{11} + q_{22} + 2(q_{11}q_{22} - q_{12}^2)^{1/2}} \tag{2}$$

where we have used a definition of ellipticity $|e| = (1-r)(1+r)^{-1}$, where r is the ratio of minor to major axes of the ellipse. For the weighted size we have a similar expression

$$R^2 = q_{11} + q_{22}. \tag{3}$$

We can calculate the variance between the ellipticity of the model and true PSF $\sigma^2(e) \equiv \langle (e - e_{\text{PSF}}^{\text{t}})^2 \rangle$ and similarly for the size $\sigma^2(R) \equiv \langle (R - R_{\text{PSF}}^{\text{t}})^2 \rangle$. Submissions were scored in real-time on a leaderboard that displayed the metric $P \equiv \frac{1}{\frac{1}{2}\langle \sigma^2(R)+\sigma^2(e)\rangle}$ where the average was taken over images in a set but not over objects asked-star positions, such that a mean variance of 10^{-3} in both ellipticity and size would have $P \sim 1.0$.

The P metric, whilst indicatively ranking the methods, does not offer any insight into the performance of a method on ellipticity and size reconstruction. In this paper we will present quantities that relate to the principal properties of the PSF more directly. These are the standard deviation of mean of the residuals of the ellipticity $\sigma(e)$ and size-squared $\sigma(R^2)/R^2$ over all asked-stars i.e. we compute the error on the mean of the residuals (the sample variance computed

Table 1: The properties of each set of images. Details are described in Section 2.1. Each category allows a different test: PSF Size allows us to test under-sampling; Atmosphere tests ground-based exposure time dependence; N_{Stars} tests spatial sampling; Mask tests telescope structure dependence; PSF-Type tests the impact of high spatial frequencies in the PSF profile vs smooth profiles; Telescope variation allows us to test the impact of three typical distortions found in data. The set order was semi-random so as to prevent participants exploiting any pattern in the set numbering. We label the fiducial sets for the Moffat and Airy profiles.

Set	Atmosphere	PSF-Type	Mask	N_{Stars}	PSF Size/pixels	Telescope Variation
1 (fid. Airy)	No	Airy	None	1000	3	None
2	No	Airy	+	1000	3	None
3	No	Airy	∗	1000	3	None
4	No	Airy	None	2000	3	None
5	No	Airy	None	500	3	None
6	No	Airy	None	1000	1.5	None
7	No	Airy	None	1000	6	None
8 (fid. Moffat)	No	Moffat	None	1000	3	None
9	Yes	Airy	None	1000	3	None
10	Yes	Moffat	+	1000	3	None
11	Yes	Moffat	∗	1000	3	None
12	Yes	Moffat	None	2000	3	None
13	Yes	Moffat	None	500	3	None
14	Yes	Moffat	None	1000	1.5	None
15	Yes	Moffat	None	1000	6	None
16	No	Airy	None	1000	3	astigmatism a_0
17	Yes	Moffat	None	1000	3	astigmatism a_0
18	No	Airy	None	1000	3	de-focus d_0
19	Yes	Moffat	None	1000	3	de-focus d_0
20	No	Airy	None	1000	3	coma c_0
21	Yes	Moffat	None	1000	3	coma c_0
22	No	Moffat	+	1000	3	None
23	No	Moffat	∗	1000	3	None
24	No	Moffat	None	2000	3	None
25	No	Moffat	None	500	3	None
26	No	Moffat	None	1000	1.5	None

using centred second order moments). We assume that any mean bias could be removed through cross-validation, in this sense it is a generous analysis to those methods with a mean residual. We average these quantities over the 50 images in each set, but in fact for all methods we find that the fractional error between images in a set is $\lesssim 10\%$.

3. Results

In total 30 submissions were made from 9 teams. As a baseline benchmark, a method in which all stars were simply stacked in an image was created, where no spatial variation in the reconstructed stars was present. Several methods generated low scores due to misunderstanding of simulation details, resulting in scores below the benchmark, and in this paper we summarise only those not affected by these issues. In the following we choose the best performing submission, for size, for each of the 12 distinct method entries. All of the submitted methods are described in Appendix A. We show the results on the fiducial Airy set (set 1 in Table 1) and the fiducial Moffat set (set 8 in Table 1) in Tables 2 and 3 respectively. In Figures 1 and 2 we present general behaviours of methods over the sets as categories were change, but for a quantitative presentation of each method we refer the reader to Figures 3 to 7 where we show pictographic tables of results.

Table 2: The results for ellipticity and size-squared on set 1 (the fiducial Airy set) for each method tested in this paper.

Method Name	$1/\sigma(e)$	$\sigma(e)/10^{-4}$	$1/[\sigma(R^2)/R^2]$	$[\sigma(R^2)/R^2]/10^{-3}$
B-Splines	3953	2.53	1348	0.742
IDW	3448	2.90	1212	0.825
RBF	3155	3.17	1259	0.794
RBF-thin	2985	3.35	1258	0.795
Kriging	1049	9.53	490	2.042
Gaussianlets	1473	6.79	392	2.548
IDW Stk	1058	9.45	277	3.604
PSFEx	1279	7.82	378	2.647
Shapelets	1256	7.96	379	2.642
PCA+Kriging	1339	7.47	314	3.180
MoffatGP	2545	3.93	429	2.331
Stacking	1441	6.94	309	3.237

Table 3: The results for ellipticity and size-squared on set 8 (the fiducial Moffat set) for each method tested in this paper.

Method Name	$1/\sigma(e)$	$\sigma(e)/10^{-4}$	$1/[\sigma(R^2)/R^2]$	$[\sigma(R^2)/R^2]/10^{-3}$
B-Splines	3690	2.71	1406	0.711
IDW	3215	3.11	1309	0.764
RBF	2967	3.37	1167	0.857
RBF-thin	2809	3.56	1163	0.860
Kriging	1477	6.77	645	1.551
Gaussianlets	2041	4.90	476	2.099
IDW Stk	1250	8	362	2.759
PSFEx	610	16.40	296	3.374
Shapelets	1931	5.18	696	1.436
PCA+Kriging	1161	8.61	351	2.853
MoffatGP	2857	3.50	139	7.209
Stacking	1259	7.94	309	3.236

Overall we find that the B-Splines, IDW and RBF methods reconstruct the ellipticity and size most accurately (see Gentile et al., 2012), with $\sigma(e) \approx 2.5\mathrm{x}10^{-4}$ and $\sigma(R^2)/R^2 \approx 7.4\mathrm{x}10^{-4}$ over all sets[4]. We note however that this is a snapshot of performance and that further investigations into tunable aspects of code could result in improvements in all methods.

We summarise the behaviour of the submissions below. In each test all other parameters are kept fixed except those discussed (with fiducial values of 1000 known star positions, no mask, and telescope parameters given in Section 2). We refer to Figures 1 and 2 that show the change in the inverse variance of the reconstructed PSFs over the fiducial sets (set 1 for Airy, and set 8 for Moffat profiles, see Table 2) when each of the categories is varied. In Figures 3 to 7 we show pictographic tables of results.

- **PSF Type**. For the best performing methods we find a trend that both ellipticity and size are estimated more accurately for the Airy PSF than for the the Moffat PSF.

- **Addition of Kolmogorov Power**. For each set combination where both Moffat and Moffat-plus-Kolmogorov power are available (e.g. the 4-arm + masks) we find evidence for methods performing less well with the addition of Kolmogorov power (see also Figures 3, 4, 5). In Figure 6 we also show the impact of adding a Kolmogorov power spectrum to a set that uses an Airy PSF profile. We find that the addition of this random component degrades the residual ellipticity reconstruction by a factor of $\gtrsim 2 - 5$, but has less impact on size reconstruction, as expected since the power is in ellipticity only. These results will necessarily depend on the amplitude of the assumed power spectrum, this will vary for each ground-based telescope, and knowledge/information about this is improving (e.g. Heymans et al. 2012). In addition atmospheric turbulence also changes the PSF size, but we do not simulate this here. It is possible that, depending on the site and weather, the impact of turbulence may be weaker or stronger than that simulated for this study.

- **Masks**. We show results for the mask variation in Figure 3. We find that for all methods the presence of diffraction spikes does not degrade the ability to measure the ellipticity of the PSF. For the Airy function the diffraction spikes act to increase the effective size of the PSF, this enables methods to measure the fractional error $\sigma(R^2)/R^2$ more accurately; but note that for a fixed $\sigma(R^2)$ a large size will decrease the fractional error by definition. For the Moffat PSF the diffraction spikes impact the size estimation significantly. We note however this was a simple addition of a mask with no commensurate change in the variation of ellipticity or size across a field of view, also the diffraction spikes contained low flux (only observable with the eye if one stacked all stars) higher signal-to-noise stars would change this, we leave an investigation of these effects for future work.

- **Number of Stars**. We find that all methods are only weakly dependent, or insensitive to the number of stars used to reconstruct the PSF in these simulations, except for those sets in which we include a Kolmogorov power spectrum where we find that a larger number of stars results in a better reconstruction for the best methods (see Figure 4). This indicates that PSFs with spatial power on smaller scales require more stars for a particular reconstruction accuracy than PSFs without power on small spatial scales.

- **Size of PSF** For the Airy profile we find that the larger the PSF the more accurately its size can be measured, for the Moffat we find a weak dependence with size. This

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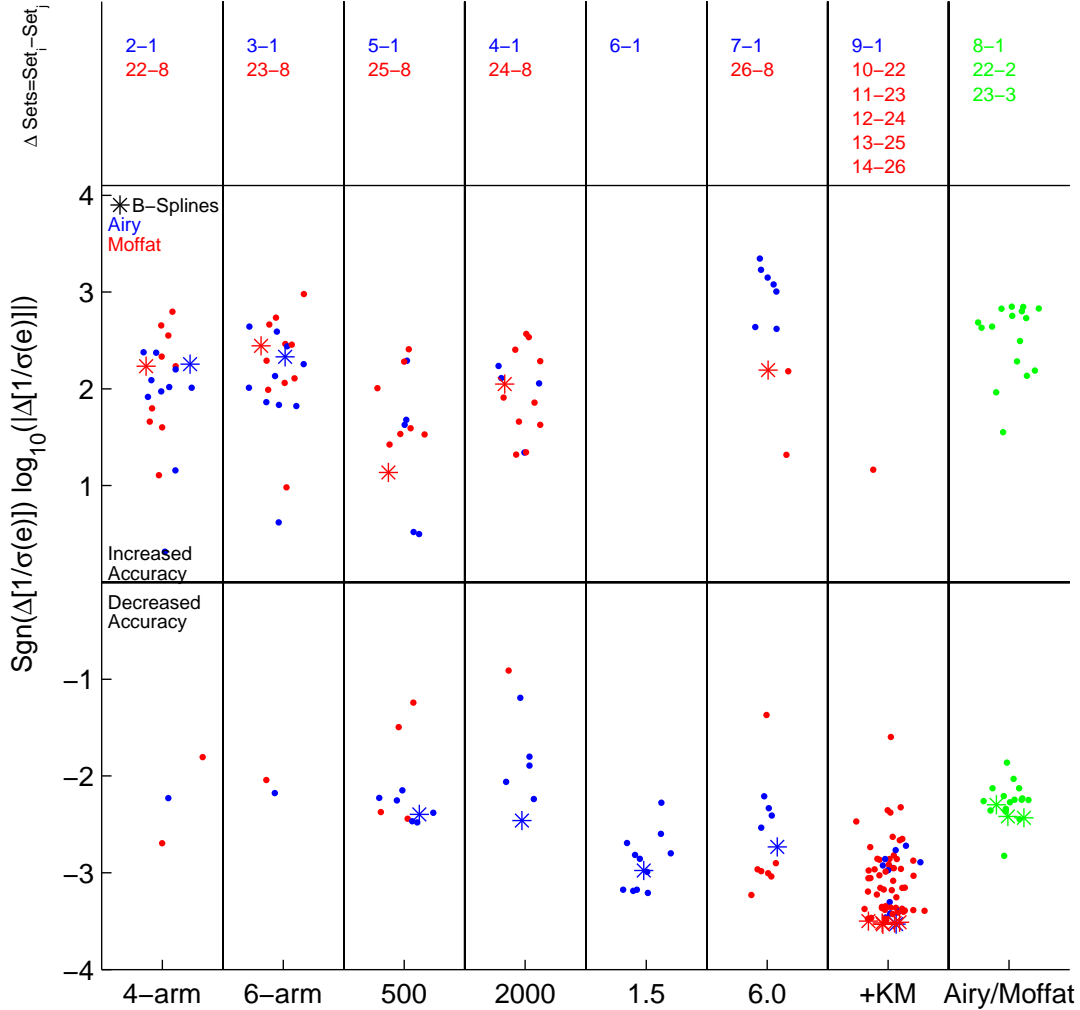[4]B-Splines also achieved the highest leaderboard P value.

Fig. 1.— The change in the inverse variance in the residual ellipticity for each method for each category varied. The sets used in differencing the categories are shown in the upper panels (Set_i-Set_j), and we refer the reader to Table 1. Each point represents a method, the stars represent method B-Spline, points within a bin are randomised within an x-bin for clarity. The log of the change is shown, with the sign preserved (i.e. $sgn[x]\log_{10}[|x|]$ where $x = (1/\sigma(e)_{\mathrm{fiducial}}) - (1/\sigma(e))$) so that negative values represent a decrease in accuracy and positive values an increase in accuracy. The first seven vertical panels show changes for the Moffat (red) and Airy profile (blue), the rightmost panel shows the change in accuracy when the profile is changed from Airy to Moffat but all other aspects of the PSF at kept the same. The parameters varied are the mask (4-arm or 6-arm; changed from no mask), number of stars (500 or 2000; changed from 1000), PSF size (1.5 or 6.0 pixels; changed from 3.0 pixels) and the addition of Kolmogorov power in ellipticity.

Fig. 2.— The change in the inverse variance in the residual size-squared for each method for each category varied. The sets used in differencing the categories are shown in the upper panels (Set$_i$-Set$_j$), and we refer the reader to Table 1. Each point represents a method, the stars represent method B-Spline, points within a bin are randomised in within an x-bin for clarity. The log of the change is shown, with the sign preserved (i.e. $sgn[x]\log_{10}[|x|]$ where $x = (1/[\sigma(R^2)/R^2]_{\rm fiducial}) - (1/[\sigma(R^2)/R^2]))$ so that negative values represent a decrease in accuracy and positive values an increase in accuracy. The first seven vertical panels show changes for the Moffat (red) and Airy profile (blue), the rightmost panel shows the change in accuracy when the profile is changed from Airy to Moffat but all other aspects of the PSF at kept the same. The parameters varied are the mask (4-arm or 6-arm; changed from no mask), number of stars (500 or 2000; changed from 1000), PSF size (1.5 or 6.0 pixels; changed from 3.0 pixels) and the addition of Kolmogorov power in ellipticity.
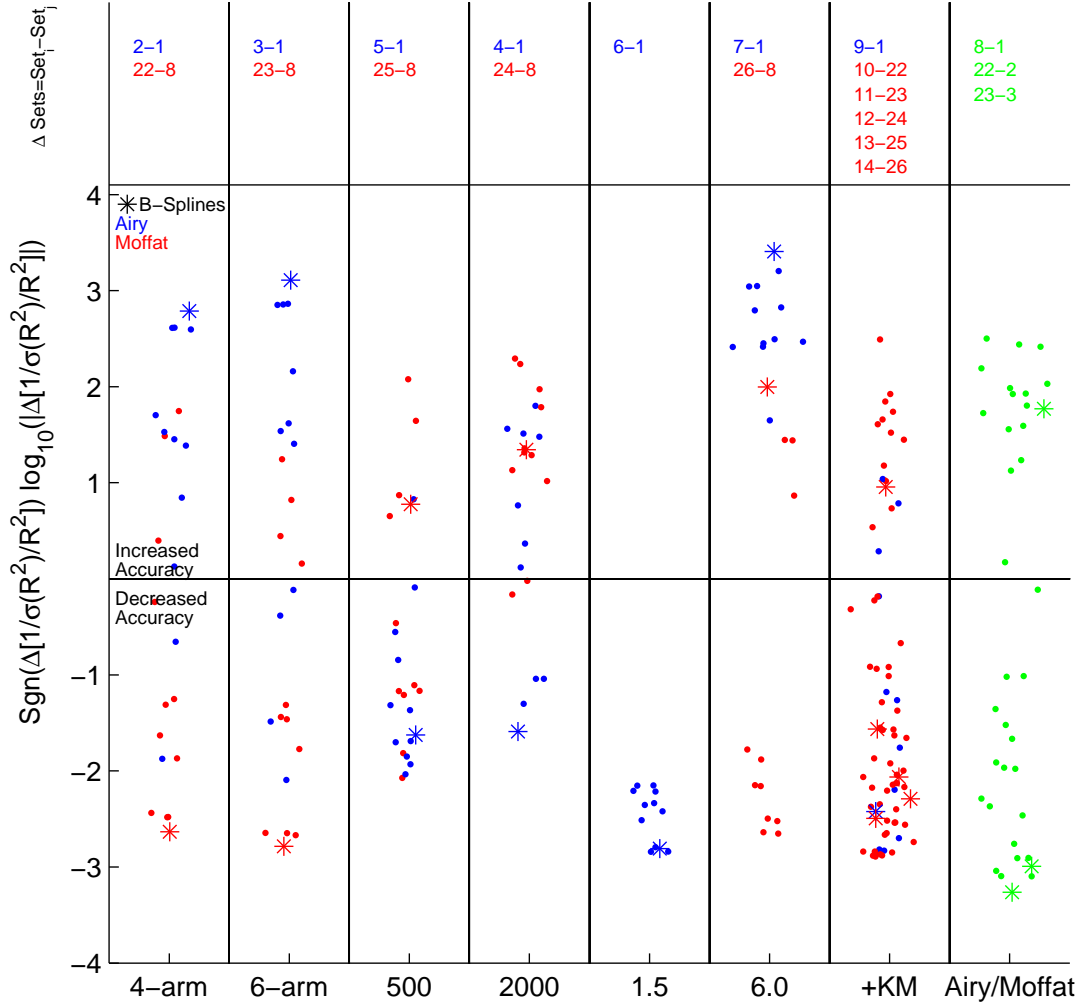
is understandable because a larger PSF is better sampled and hence the size is easier to measure. However we stress that an increase of the size of the PSF relative to the apparent size of galaxies will cause the galaxies to be less well-resolved, losing information and placing greater demands on shape measurement (Paulin-Henriksson et al. 2008). Also with the simulations presented the impact of sampling on weak lensing shape measurement was not tested, only the performances of the PSF interpolation methods. We show results for the PSF size variation in Figure 5. When trading requirements of PSF model residuals against requirements for resolution (i.e. the absolute size of the ellipticity and PSF) such behaviour should be noted.

• **Telescope Parameters**. We show results for the PSF size variation in Figure 7. In varying the telescope parameters in the Jarvis et al. (2008) model we change the fiducial parameters respectively ($a_0 = 0.014$, $d_0 = -0.006$, $c_0 = -0.010$), to $a_0 = -0.011$, $d_0 = 0.009$ and $c_0 = -0.011$ i.e. an opposite astigmatism, a positive de-focus and a 10% increased coma. We find that methods in this experiment were not affected by the change in defocus, but performed better with the change in these astigmatism and coma parameters.

We discuss each method individually in Appendix A.

Figure 3 — left panel: $1/\sigma(e)$ (key: 10^4, 10^3, 10^2; Moffat+KM green, Airy blue, Moffat red)

Method	No Mask KM	No Mask Airy	No Mask Moffat	4–arm KM	4–arm Airy	4–arm Moffat	6–arm KM	6–arm Airy	6–arm Moffat
B–Splines		3953	3690	561	4132	3861	557	4167	3968
IDW		3448	3215	575	3571	3571	572	3584	3676
RBF		3155	2967	506	3257	3030	507	3257	3096
RBF–thin		2985	2809	489	3067	2849	491	3058	2924
Kriging		1049	1477	555	1208	1692	555	1325	1764
Gaussianlets		1473	2041	532	1577	2212	534	1653	2331
IDW Stk		1058	1250	544	889	755	546	908	1445
PSFEx		1279	610	415	1515	1236	487	1669	1562
Shapelets		1256	1931	540	1271	1976	540	1325	2028
PCA+Kriging		1339	1161	553	1577	1613	575	1779	1706
MoffatGP		2545	2857	541	2639	2793	536	2611	2747
Stacking		1441	1259	552	1443	1272	547	1445	1269

Figure 3 — right panel: $1/[\sigma(R^2)/R^2]$

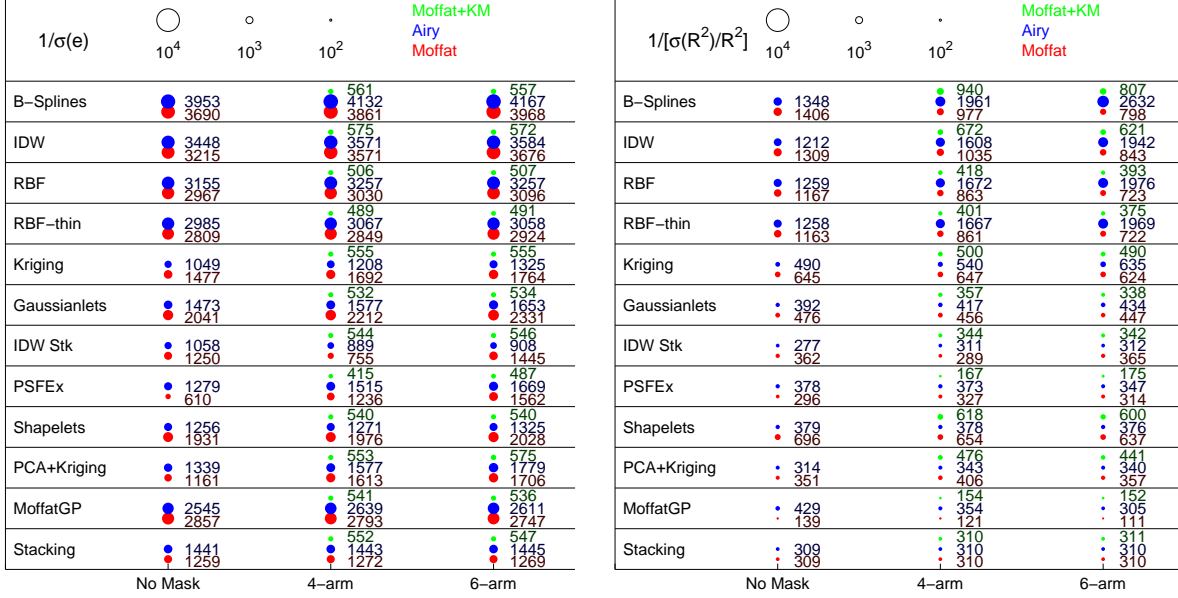| Method	No Mask KM	No Mask Airy	No Mask Moffat	4–arm KM	4–arm Airy	4–arm Moffat	6–arm KM	6–arm Airy	6–arm Moffat
B–Splines		1348	1406	940	1961	977	807	2632	798
IDW		1212	1309	672	1608	1035	621	1942	843
RBF		1259	1167	418	1672	863	393	1976	723
RBF–thin		1258	1163	401	1667	861	375	1969	722
Kriging		490	645	500	540	647	490	635	624
Gaussianlets		392	476	357	417	456	338	434	447
IDW Stk		277	362	344	311	289	342	312	365
PSFEx		378	296	167	373	327	175	347	314
Shapelets		379	696	618	378	654	600	376	637
PCA+Kriging		314	351	476	343	406	441	340	357
MoffatGP		429	139	154	354	121	152	305	111
Stacking		309	309	310	310	310	311	310	310

Fig. 3.— The inverse variance in the residual ellipticity and size-squared for each method (horizontal panels) for the three mask cases (no mask, 4-arm + and 6-arm ∗) for the Moffat-plus-Kolmogorov case (green), Moffat with no Kolmogorov (red), and the Airy (blue) profile. The circles represent the inverse variance of the residual ellipticity and size-squared where the area scales in proportion to these parameters and the numbers are given next to each circle; a key is given in the top panel. Where no number/circle is provided there was no set for this combination of PSF type and mask type. Fractional errors on the inverse variances are ≈ 10% for all methods.

Figure 4 — left panel: $1/\sigma(e)$ (columns 500, 1000, 2000)

Method	500 KM	500 Airy	500 Moffat	1000 Airy	1000 Moffat	2000 KM	2000 Airy	2000 Moffat
B–Splines	553	3704	3704	3953	3690	560	3663	3802
IDW	552	3155	3472	3448	3215	601	3448	3584
RBF	478	2976	2950	3155	2967	537	3040	3040
RBF–thin	461	2817	2778	2985	2809	520	2907	2890
Kriging	528	1053	1504	1049	1477	578	1049	1499
Gaussianlets	510	1515	2075	1473	2041	567	1495	2083
IDW Stk	544	1106	1441	1058	1250	591	1188	1504
PSFEx	135	1038	374	1279	610	261	1393	803
Shapelets	535	1259	1965	1256	1931	541	1241	1976
PCA+Kriging	458	1036	884	1339	1161	558	1511	1504
MoffatGP	539	2740	2959	2545	2857	541	2481	2849
Stacking	549	1300	1299	1441	1259	547	1267	1280

Figure 4 — right panel: $1/[\sigma(R^2)/R^2]$ (columns 500, 1000, 2000)

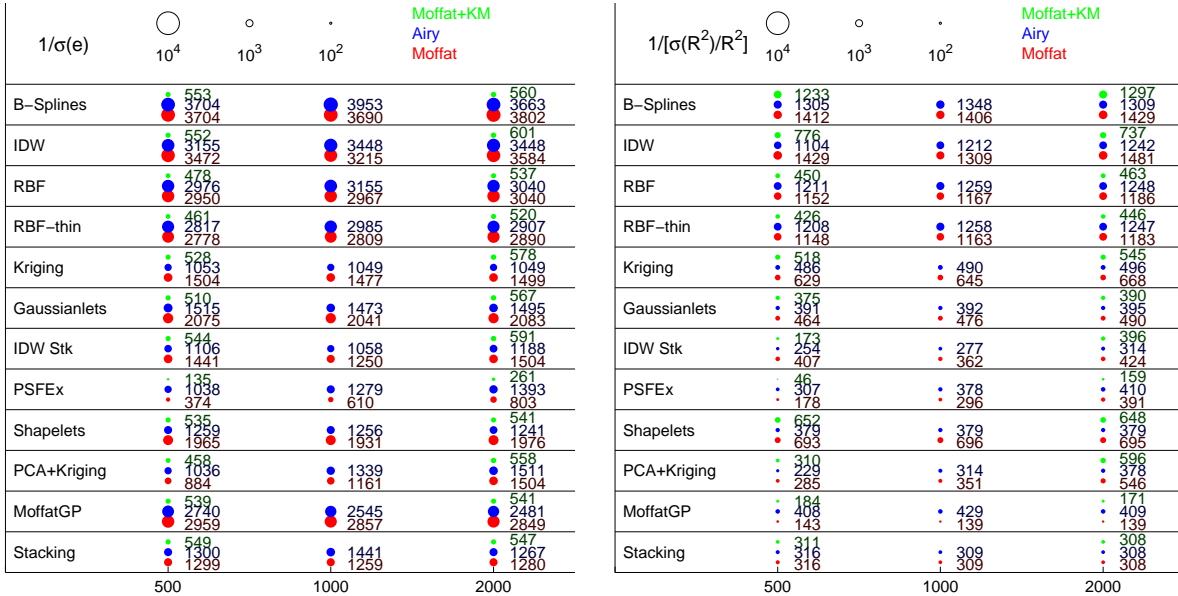| Method	500 KM	500 Airy	500 Moffat	1000 Airy	1000 Moffat	2000 KM	2000 Airy	2000 Moffat
B–Splines	1233	1305	1412	1348	1406	1297	1309	1429
IDW	776	1104	1429	1212	1309	737	1242	1481
RBF	450	1211	1152	1259	1167	463	1248	1186
RBF–thin	426	1208	1148	1258	1163	446	1247	1183
Kriging	518	489	629	490	645	545	496	668
Gaussianlets	375	397	464	392	476	390	395	490
IDW Stk	173	254	407	277	362	396	314	424
PSFEx	46	307	178	378	296	159	410	391
Shapelets	652	379	693	379	696	648	379	695
PCA+Kriging	310	229	285	314	351	596	378	546
MoffatGP	184	408	143	429	139	171	409	139
Stacking	311	316	316	309	309	308	308	308

Fig. 4.— The inverse variance in the residual ellipticity and size-squared for each method (horizontal panels) for the three known-star number cases (500, 1000, 2000 stars) for the Moffat-plus-Kolmogorov case (green), Moffat with no Kolmogorov (red), and the Airy (blue) profile. The circles represent the inverse variance of the residual ellipticity and size-squared where the area scales in proportion to these parameters and the numbers are given next to each circle; a key is given in the top panel. Where no number/circle is provided there was no set for this combination of PSF type and number of stars. Fractional errors on the inverse variances are ≈ 10% for all methods.
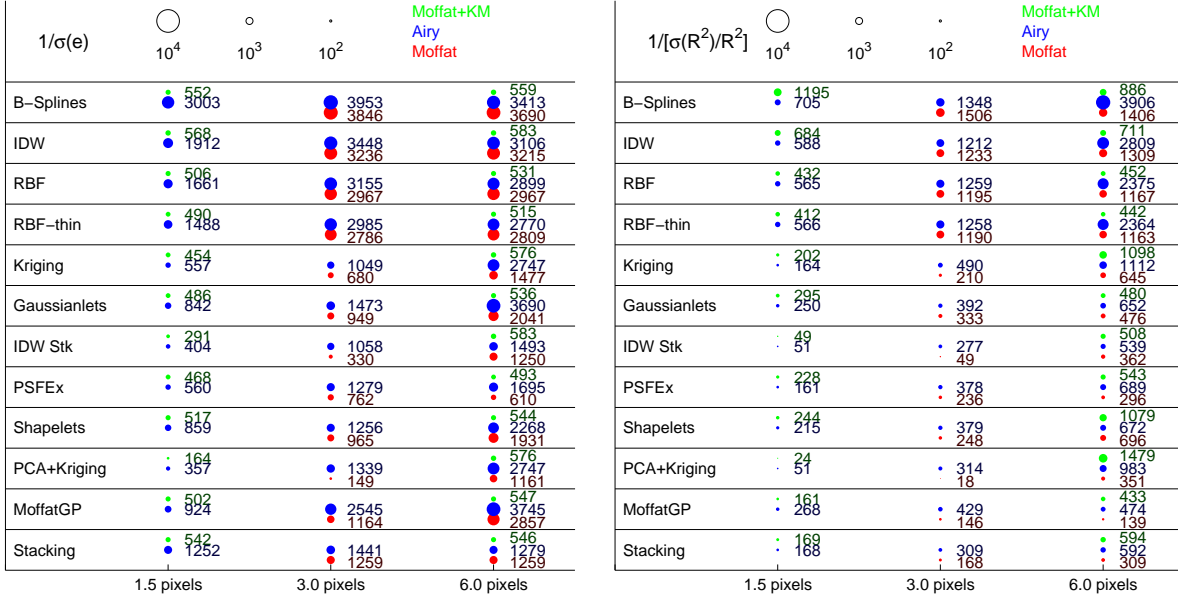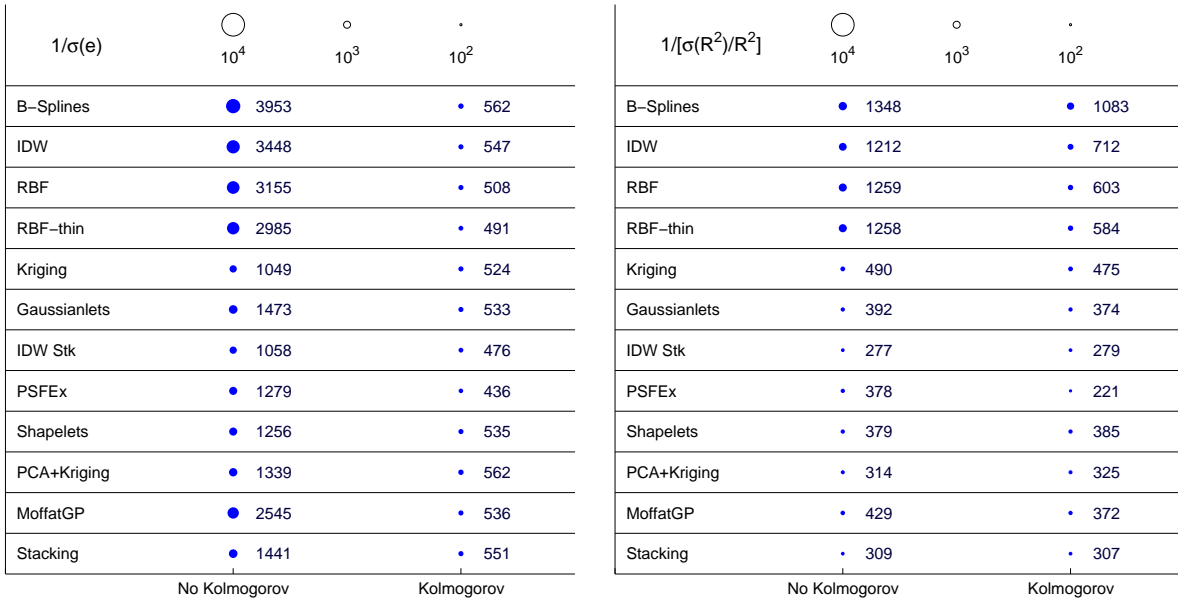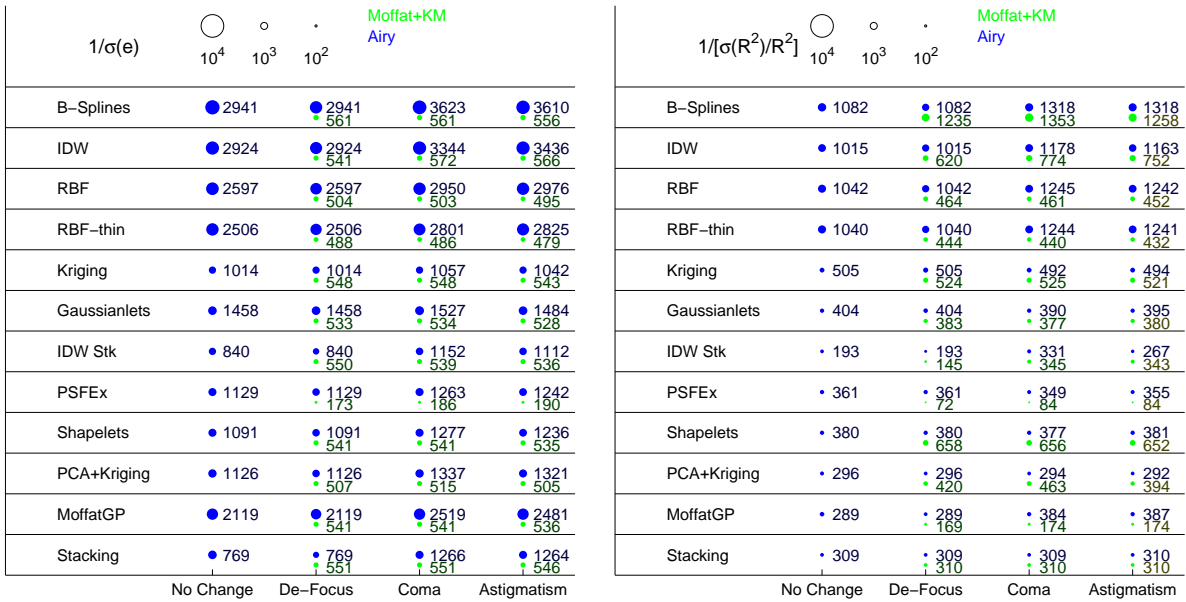
Fig. 5.— The inverse variance in the residual ellipticity and size-squared for each method (horizontal panels) for the three PSF size cases (1.5, 3.0 and 6.0 pixels) for the Moffat-plus-Kolmogorov case (green), Moffat with no Kolmogorov (red), and the Airy (blue) profile. The circles represent the inverse variance of the residual ellipticity and size-squared where the area scales in proportion to these parameters and the numbers are given next to each circle; a key is given in the top panel. Where no number/circle is provided there was no set for this combination of PSF type and PSF size. Fractional errors on the inverse variances are $\approx 10\%$ for all methods.

Fig. 6.— The inverse variance in the residual ellipticity and size-squared for each method (horizontal panels) for the two cases where a Kolmogorov power is added, or not to a set with a Airy (blue) profile. The circles represent the inverse variance of the residual ellipticity and size-squared where the area scales in proportion to these parameters and the numbers are given next to each circle; a key is given in the top panel. Fractional errors on the inverse variances are $\approx 10\%$ for all methods.

1/σ(e)	No Change	De-Focus	Coma	Astigmatism
B–Splines	2941	2941 / 561	3623 / 561	3610 / 556
IDW	2924	2924 / 541	3344 / 572	3436 / 566
RBF	2597	2597 / 504	2950 / 503	2976 / 495
RBF–thin	2506	2506 / 488	2801 / 486	2825 / 479
Kriging	1014	1014 / 548	1057 / 548	1042 / 543
Gaussianlets	1458	1458 / 533	1527 / 534	1484 / 528
IDW Stk	840	840 / 550	1152 / 539	1112 / 536
PSFEx	1129	1129 / 173	1263 / 186	1242 / 190
Shapelets	1091	1091 / 541	1277 / 541	1236 / 535
PCA+Kriging	1126	1126 / 507	1337 / 515	1321 / 505
MoffatGP	2119	2119 / 541	2519 / 541	2481 / 536
Stacking	769	769 / 551	1266 / 551	1264 / 546

$1/[\sigma(R^2)/R^2]$	No Change	De-Focus	Coma	Astigmatism
B–Splines	1082	1082 / 1235	1318 / 1353	1318 / 1258
IDW	1015	1015 / 620	1178 / 774	1163 / 752
RBF	1042	1042 / 464	1245 / 461	1242 / 452
RBF–thin	1040	1040 / 444	1244 / 440	1241 / 432
Kriging	505	505 / 524	492 / 525	494 / 521
Gaussianlets	404	404 / 383	390 / 377	395 / 380
IDW Stk	193	193 / 145	331 / 345	267 / 343
PSFEx	361	361 / 72	349 / 84	355 / 84
Shapelets	380	380 / 658	377 / 656	381 / 652
PCA+Kriging	296	296 / 420	294 / 463	292 / 394
MoffatGP	289	289 / 169	384 / 174	387 / 174
Stacking	309	309 / 310	309 / 310	310 / 310

Key (top panel): circle sizes 10^4, 10^3, 10^2; Moffat+KM (green); Airy (blue).

Fig. 7.— The inverse variance in the residual ellipticity and size-squared for each method (horizontal panels) for the cases where the telescope parameters are varied for the Airy (blue) profile and the Moffat-plus-Kolmogorov profile (green). The circles represent the inverse variance of the residual ellipticity and size-squared where the area scales in proportion to these parameters and the numbers are given next to each circle; a key is given in the top panel. Where no number/circle is provided there was no set for this combination of PSF type and telescope parameter. Fractional errors on the inverse variances are ≈ 10% for all methods.

4. Conclusions

This paper presents the first blind simulation challenge aimed to test optical PSF reconstruction methods. Simulations were generated in which participants were presented with a spatially varying PSF, sparsely sampled by stars, and asked to reconstruct the PSF at non-star positions. The competition, the GREAT10 Star Challenge, attracted 30 submissions from 9 teams; several of these teams were from non-astronomy backgrounds. The simulation presented participants with 27,500 stars over 1300 images subdivided into 26 sets, where in each set a category change was made in the type or spatial variation of the PSF. The simulations were intentionally simplistic, so as to present the problem in an approachable way; in particular the spatial variation of the PSF and the form of the PSF use simple analytic functions. In addition only spatial variation, not temporal variation, was tested; hence these results should not be used to make specific statements about any particular experiment but should provide a benchmark with which methods can be tested and improved[5]

In this paper we analyse the submissions by testing how well each one can measure the ellipticity and size of the PSF. We quantify this as the inverse variance in the modelled PSF in each image for ellipticity and sized-squared – defined using weighted quadrupole moments. This study was motivated by a desire to find methods that will be of use for weak gravitational lensing, where the PSF must be reconstructed to high accuracy (Paulin-Henriksson et al. 2008, 2009) at galaxy positions, but these results should also be of more general interest for any science case that analyses galaxy images with optical data.

The submissions, and this paper, present a snapshot of any methods' ability to model the PSF. Due to the nature of the competitive blind submissions post-challenge tuning of methods, that may yield significant improvements for any given method over the results presented here (see Gentile et al., 2012 for example), were not investigated. Each method submitted is summarised in Appendix A. We can however make some general statements about regimes in which methods tend to perform well or poorly when run in a blind way.

The functional form of the PSF was either a Moffat function or a Airy function, the spatial variation of the PSF was modelled using the analytic function given in Jarvis et al. (2008), in addition we optionally included diffraction spikes (+ or ∗ forms), changed the PSF size (from 3.0 pixels to 1.5 or 6.0 pixels), changed the number of stars (from 1000 to 500 or 2000), and added an atmospheric turbulence pattern in ellipticity (with a Kolmogorov power spectrum). To summarise the conclusions we find that

- The best methods can reconstruct the PSF with an accuracy of $\sigma(e) \approx 2.5 \times 10^{-4}$ and $\sigma(R^2)/R^2 \approx 7.4 \times 10^{-4}$ over all sets.

- Methods that performed poorly did so in part because the functional form of the PSF was not modelled correctly (in particular the Airy function).

- Smaller PSFs were more difficult to model than larger PSFs for the Airy function. But we add a caution that this does not mean larger PSFs are better for weak lensing, because information on a target object is lost; instead this means that well sampled PSFs are better for weak lensing.

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[5]Data is available for download here http://great.roe.ac.uk/data/solutions/.

- Diffraction spikes caused the size of Moffat PSFs to be modelled less accurately, but Airy PSFs more accurately, due to the increase in the effective size.

- The addition of atmospheric Kolmogorov power (equivalent to short exposure PSFs, see Heymans et al., 2012) made ellipticity and size reconstruction less accurate by a factor of $\gtrsim 2 - 5$ for all methods. We add the caveat that the temporal nature of varying PSFs was not investigated, therefore methods such as cross-correlation between sequential images, that could potentially improve modelling, were not investigated.

For subsequent blind PSF modelling challenges the realism of the temporal and wavelength dependent nature of PSF variation could be included, and the simulations could be tailored to specific experiments.

Modelling the PSF is of critical importance in efforts to understand the nature of dark energy and dark matter using weak gravitational lensing; where any inaccuracy in the modelled PSF can cause biases, and increased errors of cosmological parameters of interest. To address this crucial open problem this initial presentation of a blind PSF reconstruction challenge will hopefully provide a benchmark upon which methods can continue to be refined and tested.

Contributions: All authors contributed to the writing and analysis presented. TDK was PI of GREAT10, defined and created the simulations, and lead the analysis. TDK, BR, MG, CH, RM were active members of the GREAT10 team during (12/2010 to 09/2011) and after the challenge. BR created the FITS image simulation code. MGe, FC, GM, DG, MK, KG, AS, AM, GL, BX submitted entries to the GREAT10 star challenge. DW maintained the GREAT10 leaderboard and processed submissions with TDK during the challenge.

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Appendix A: Description of Methods

Here we include a brief description and references for each of the methods submitted to the challenge.

Several methods use the name "Kriging", which is in fact the same method as Gaussian process regression, the method submitted for the methods MoffatGP; Kriging is a different term which has been in use in the geostatistics field but all are types of Gaussian processes.

PSFEx (Gruen)

PSFEx uses version 3.9.1 of the PSFEx software (Bertin 2011). The method models the PSF using a functional basis, the coefficients of which are allowed to vary with a polynomial dependence on the position in the field. Details of the configuration can be found in the PSFEx manual[6]. For the GREAT10 submission, the functional basis is chosen to be a sub-pixel grid, from which PSF images on the input pixel scale are produced using Lanczos interpolation of order 4. In order to improve configuration parameters, the P metric is calculated on stars reserved from the fit. For this a Gaussian weight function with much smaller scale than in the final analysis (3pix FWHM) is used in order to suppress the noise in the images. The spatial variation was chosen to be of order 8 (4) on the sets with (without) atmospheric Kolmogorov power and the size of the sub-pixel grid to be 1/4.7 of the PSF FWHM on all sets except the very undersampled sets 6, 14 and 26, where a scale of 0.25pix is used instead. Note that these choices were made without knowledge of the true properties of the sets.

PCA+Kriging (Li, Xin)

The basic idea is to find the principal components of ensemble of stars in an image. To find the right principal components (PCs), were needed to align all of the stars at a same center. Here a fast algorithm (Li et al. 2012) was used to locate the centroid for each star and then ordinary Kriging fitting algorithm was used to reproduce the star, whose center was exactly located at the center of stamp grid. Each star was represented by 5 PCs, with five corresponding coefficients. According to the noise in the star stamp, an additional Gaussian noise component was included in each pixel and re-evaluated the corresponding coefficients in ten realisations; this helped us to estimate the uncertainties for each coefficient of PCs and each star. Ordinary Kriging fitting process was the used to predict the value of each coefficient at the asked positions and the new stars were composed of these fives PCs with the predicted coefficients.

Gaussianlets (Li, Xin)

Gaussianlets is a simplified version of shapelets without any angular components, i.e., there are no shapelets with $m \neq 0$. The ellipticity of each star was calculated at the first step, then the size of each star R_i was estimated quickly using the algorithm described in Li et al. (2012). One set of unique gaussianlets with a maximum order $n_{\max} = 4$ were created with $R = \langle R_i \rangle$. These gaussianlets are circularly symmetric and were then reshaped into elliptical

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profiles according to the ellipticities that were measured in the first step to fit an individual star. The coefficients of gaussianlets were calculated by minimizing a chi-square function. Finally each star was described by 7 parameters, e_1, e_2 and the five coefficients of gaussianlets. Ordinary Kriging interpolation was then used to predict these seven parameters at the asked positions. To reproduce the expected virtual star, the gaussianlets were reshaped according to e_1 and e_2 and were added up together according to their coefficients.

B-Splines (Gentile, Courbin, Meylan)

The B-Splines method, like the IDW, RBF and Kriging schemes also described in this article, uses the same underlying PSF estimation scheme that consists of the following stages. First, an elliptical Moffat profile is fitted to each star at known position. Fitting is performed using a custom-developed minimizer based on an "adaptive cyclic coordinate descent algorithm". This minimizer is also used in the *gfit* galaxy shape measurement method described in Kitching et al. (2012a). Second, an analysis is performed of the spatial distribution of each Moffat parameter across the image. Third, a spatial interpolation scheme is adopted (here B-Splines) to predict the values of each Moffat parameter p at asked positions. Finally, pixelized star images are reconstructed at asked positions, based on the interpolated Moffat profile.

B-Splines perform a spatial interpolation of individual Moffat PSF parameters using the bivariate basis-spline algorithm described in Dierckx (1980, 1995) and implemented in the Python SciPy interpolate module. The main parameters affecting the interpolation are the degree of the spline, the number of knots, and a smoothing factor. A 3rd order spline was used but the algorithm was allowed to automatically optimize the number of knots and the smoothing factor. A more thorough description of B-Splines, as well as the IDW, RBF and Kriging interpolation methods can be found in Gentile et al. (2012).

Inverse Distance Weighting (IDW) (Gentile, Courbin, Meylan)

The IDW interpolation algorithm (Shepard 2007) is used to interpolate the Moffat parameters of the fitted PSF (see B-splines). Weights are allocated to the stars or parameters to interpolate. The closer the observations from a target location, the greater the weight ascribed to them. The estimated value of the parameter at the target point is a weighted sum of the values of all neighboring observations considered. The weighting power γ determines how fast the weights tend to zero as distances increase. The Star Challenge results were obtained with $\gamma = 2$ with a neighborhood size between 5 and 15 pixels depending on the density of stars.

Radial Basis Function (RBF and RBF-thin) (Gentile, Courbin, Meylan)

The RBF and RBF-thin methods make use of Radial Basis Functions to predict the values of the PSF parameters at non-star positions. As in B-splines the PSF is approximated by a Moffat profile. A Radial Basis Function (Buhmann 2003; Press et al. 2007), is a radially-symmetric, real-valued function, whose value at a target location only depends on the distance to some other point. The prediction at a target location is based on the weighted sum of the RBFs evaluated in a neighborhood centered at that location.

The RBF and RBF-thin methods respectively use the linear and thin-plate functions. Their implementation is based on the interpolation function available in the Python SciPy interpolate module with a neighborhood size between 25 and 30 pixels. For the submission to the challenge smoothing was disabled, i.e., exact interpolation was used where the PSF reconstructed at known positions should be exactly the input data.

Kriging (Gentile, Courbin, Meylan)

Ordinary Kriging (e.g. Waller 2004; Webster & Oliver 2007) is used to interpolate PSF parameters (Moffat profiles as in B-splines) across the PSF field. For the Star Challenge, a unique implementation was created in Python for greater flexibility and control of the algorithm. In this version, no attempt was made to correct for any spatial anisotropy or drift found in the data. The experimental variograms were fitted using the Levenberg-Marquardt (Levenberg 1980; Marquardt 1963) fitting function from the SciPy optimize module. The program dynamically selects the theoretical variogram models and parameters that produce the best fit. The area used for interpolation is a circular area with a radius between 700 and 1000 pixels from the center of the 4800×4800 PSF field. Lag distances were selected in the range $100 \leq h \leq 300$ pixels depending on the image and the PSF model parameter to estimate. The number of observations N to include in the interpolation neighborhood was typically $5 \leq N \leq 20$ depending on the image star density. As a rule of thumb, a tolerance was adopted for the distances $\Delta h \approx h/2$ and angles $\Delta \theta = 22.5°$.

IDWStk (Gentile, Courbin, Meylan)

The IDWStk method experimented with an algorithm whereby the star postage stamp to reconstruct at asked position is estimated by *stacking the pixels* of nearby, surrounding stamps located at known positions. Each pixel carries a weight that depends on its distance to the location where reconstruction has to take place. These weighting factors are calculated using Inverse Distance Weighting (IDW). For the Star challenge, the number of surrounding nearby stars in the stacking was typically 10.

MoffatGP (Georgatzis, Mariglis, Storkey)

First, each 48x48 star image was reduced to a 30x30 image. Predictions for the coefficients were made at the asked positions (with their corresponding offsets) and then the star images at test positions were reconstructed using the Moffat function (generating a 48x48 image for each star patch). Five coefficients per star patch were produced and then used as training outputs for the regression method. Regression was performed using the Gaussian Process (GP) framework on an augmented input space. Along with the stars' centre locations, for each star patch a distinct variable the offset of the star centre from the bottom left corner of the centre pixel was isolated, and provided as an additional input to the GP. The neural network covariance function Rasmussen et al. (2006) was chosen to encode correlations between data points. Predictions for the coefficients were made at the asked positions (with their corresponding offsets) and then the star images at test positions were reconstructed using the Moffat model (generating a 48x48 image for each star patch). The method is described in more detail in Georgatzis (2011).